JOURNAL OF AIRCRAFT
Vol. 37, No. 3, May–June 2000

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Implications of the Insensitivity of Vortex Lift to Sweep

Lance W. Traub*
Texas A&M University, College Station, Texas 77843-3141

Nomenclature

b = wing span
C_L = lift coefficient
C_p = pressure coefficient
C_s = leading-edge suction coefficient
C_T = leading-edge thrust coefficient
c = wing root chord
c_s = sectional leading-edge suction coefficient
k, K = constant
k_i = induced efficiency constant
k_p = potential constant
k_v = vortex lift constant
S = wing area
s = local semispan
U = freestream velocity
x = chordwise direction
y = spanwise direction
y_v = spanwise location of vortex core nondimensionalized by local semispan
z_v = height of vortex core above wing surface nondimensionalized by local semispan
α = angle of attack
Γ = vortex circulation
ϵ = wing apex half-angle
Λ = leading-edge sweep angle

Subscripts

p = potential
v = vortex

Introduction

POLHAMUS's[1] leading-edge suction analogy provides an accurate and simple methodology to estimate the lift of slender delta wings. Polhamus decomposed lift into two components, one due to potential lift and the other due to vortex lift. The potential lift component is defined as the attached flow lift component in the absence of leading-edge suction and is represented by[1]

$$C_{Lp} = k_p \sin\alpha \cos^2\alpha \tag{1}$$

where k_p is the wing lift curve slope at zero lift and for delta wings may be estimated as[2]

$$k_p = 4\tan^{0.8}\epsilon \tag{2}$$

The vortex lift coefficient is described by[1]

$$C_{Lv} = k_v \sin^2\alpha \cos\alpha \tag{3}$$

where $k_v = \partial C_s/\partial \sin^2\alpha$ and is relatively invariant with Λ (see Ref. 1), the leading-edge sweep of the wing. Frequently it is assumed that $k_v = \pi$ (slender wing theory gives $k_v = \pi/\sin\Lambda$). The spanwise circulation distribution of the vortex of a thin planar delta wing with leading-edge vortex separation may be approximated as[2]

$$\Gamma(s) = \frac{1.106cUk_p\sin\alpha}{4k}\left[\frac{2s}{b}\sqrt{1-\left(\frac{2s}{b}\right)^2} + \arcsin\frac{2s}{b}\right] \tag{4a}$$

where[2] $k = 0.5$, or

$$\Gamma(s) = 4.63Uc\tan^{0.8}\epsilon\tan^{1.2}\alpha\cos\alpha(2s/b) \tag{4b}$$

Equation (4b) gives the spanwise circulation distribution of a delta wing based on an expression provided by Hemsch and Luckring[3] modified to include a spanwise conical flow distribution such that the circulation increases in a linear fashion. Visser and Nelson[4] suggested the correlating constant of 4.63. Equations (4a) and (4b) show that increasing the sweep of the wing (ϵ and k_p reducing) results in a reduction in the strength of the leading-edge vortex at any corresponding spanwise location. Furthermore, as summarized by Lowson,[5] increasing slenderness results in the vortices' vertical displacement from the wing surface increasing. Consequently, increasing leading-edge sweep both reduces the strength of the leading-edge vortices and displaces them farther away from the wing; however, as shown by Eq. (3), this appears to have no impact on the vortex lift coefficient. Why this is so is investigated.

Discussion

Equating Eq. (4a) to the Ref. 6 expression for the sectional leading-edge suction (which is equal to the local vortex lift by Polhamus's supposition) yields

$$\Gamma(s)\bigg/\left(\frac{1.106Uck_p\sin\alpha}{4k}\right) = c(s)c_s(s)\bigg/\left(\frac{SE_0C_T}{\pi\cos\Lambda}\right)$$

where $c(s)$ is the local chord. Simplification (with $E_0 = 1.106\pi/b$ and $k = 0.5$) (Ref. 2) yields

$$\frac{\Gamma(s)}{Uc(s)} = \frac{c_s(s)\cos\Lambda}{(1-k_pk_i)\sin\alpha} \tag{5}$$

where for a slender delta wing k_i may be approximated as $1/\pi AR$. Analysis of Eq. (5) shows that the invariant behavior of $c_s(s)$ with changing sweep is primarily due to the influence of the $\cos\Lambda$ term and, to a lesser extent, the effective downwash term $(1-k_pk_i)$. $U\cos\Lambda$ may be inferred as representing the freestream component perpendicular to the wing leading edge. The implications of these terms on the actual flow physics is not, however, clear, as will become apparent.

An obvious explanation for the constancy of the net vortex lift coefficient with changing sweep is the increasing influence of the leading-edge vortex on the wing as Λ increases, that is, the vortex increases in size relative to the wing span. Using Bernoulli's

Received 8 February 1999; revision received 15 December 1999; accepted for publication 20 December 1999.

*Postdoctoral Research Associate, Aerospace Engineering Department. Associate Member AIAA.

equation and for simplicity gauging the effect of the vortex strength and trajectory on the spanwise-induced loading by assuming an infinite axisymmetric line vortex give at any spanwise station, using Eq. (4b),

$$Cp_v(y) = 1 - (U_v(y)/U)^2, \qquad Cp_v(x, y) = 1 - \left[\frac{\left(4.63\tan^{1.2}\alpha\cos\alpha\cos^2\left\{\tan^{-1}(1/z_v)[(y/s) - y_v](2s/b)\right\}\right)}{2\pi z_v \tan^{0.2}\epsilon}\right]^2 \tag{6}$$

where y_v and z_v are the lateral and vertical vortex core trajectories as a fraction of the local semispan $s = x\tan\epsilon$. The $2s/b$ expression in Eq. (6) accounts for an assumed conical variation of vortex circulation with span. Note that this simplified expression ignores any chordwise velocity contribution toward upper surface pressure and, consequently, predicts too positive pressures. Using representative values for y_v and z_v from Lowson[5] and integrating spanwise and chordwise using

$$\Delta C_L = \frac{2}{S}\int_0^c\int_0^s -Cp_v(x, y)\,\mathrm{d}y\,\mathrm{d}x \tag{7}$$

show that C_L is reduced by increasing slenderness as would be expected from increased vortex displacement in combination with reduced vortex strength. Note that Eq. (7) does not account for the lower wing surface so as to reflect vortex-induced loading only. As a quantitative example, using the aforementioned values from Lowson[5] for $\alpha = 20$ deg gives the reduction in the vortex lift coefficient resulting from increasing Λ from 70 to 80 deg as 8%. This change in the local vortex lift coefficient is far greater than the variation of k_v with sweep[1] (for this example the change is approximately 0.95%). Although this example is extremely simplistic, results by Roos and Kegelman[7] for the spanwise loading over the apex region of a 60- and 70-deg sweep delta wing at $\alpha = 20$ deg show that the two wings share similar loading inboard, whereas the lesser swept wing ($\Lambda = 60$ deg) has higher peak loading under the vortex for a similar Reynolds number. Note that this result applies to the apex region of the wings as the $\Lambda = 60$ deg delta was affected by vortex breakdown over the rear of its surface.

Consequently, Roos and Kegelman's[7] results suggest that the relative increase in the size of the leading-edge vortices with increasing sweep does not influence the wing's flowfield to such an extent as to compensate for the reduction in circulation and altered trajectory. Thus, the $\cos\Lambda$ term must relate to other properties. Slenderness or sweep affects the upstream penetration of the Kutta condition, that is, a more slender wing has reduced trailing-edge influence. Also, as shown by Kirkpatrick,[8] trailing-edge effects impact the nonlinear lift to a greater extent than the linear lift. Changes in trailing-edge influence will affect both the level of vortex lift and of potential lift developed. Trailing-edge influences should be evident in the location of the wing's aerodynamic center (a.c.). If the $\cos\Lambda$ term effectively relates to the Kutta condition, it should then be possible to relate it to the location of the wing's a.c.

Slender wing theory, which precludes wake and trailing-edge effects, predicts that the location of the wing's a.c. should be at $\frac{2}{3}$ of the delta wing root chord. This may be considered an upper (or rearward) bound because the wing sweep tends to 90 deg and the wing's trailing-edge extent tends to 0. It would be expected that a natural formulation relating the a.c. and $\cos\Lambda$ would encompass some deviation from the slender wing theory a.c. location as a function of $\cos\Lambda$. It was found (empirically) that this relation is, in fact, extremely simple and is given by

$$\left(\tfrac{2}{3} - \text{a.c.}/c\right)(1/\cos\Lambda) = \text{const} \tag{8}$$

Equation (8) was determined using both experimentally[9] and numerically[10] determined values for the a.c. location of delta wings at low α. As may be seen in Fig. 1, this simple expression accurately correlates the relationship of the wing's a.c. to its leading-edge sweep. The a.c. data from the vortex-lattice method was taken as the average of the potential flow and vortex lift contributions to be consistent with the experimental data, which contained both contributions. Computationally,[10] at low α, the potential and vortex lift a.c. locations are within 0.2% of each other for $\Lambda = 65$ deg and slowly diverge to within 3.1% for $\Lambda = 80$ deg. As seen, the data collapse to an average value of approximately 0.205 for the experimental data and 0.176 for the numerical data. Consequently, this yields a simple expression for the a.c. location of a delta wing at low α.

$$\text{a.c.}/c = \tfrac{2}{3} - K\cos\Lambda \tag{9}$$

where $K \approx 0.2$ for a thin planar delta wing. For delta wings, increasing α generally sees a forward migration of the wing's a.c. due to trailing-edge effects. An empirical correction to account for this effect was determined to be of the form

$$\text{a.c.}/c = \tfrac{2}{3} - K(1 + 0.04\alpha)\cos\Lambda \tag{10}$$

with α measured in degrees. Figure 2 shows a comparison of the experimental data from Ref. 9 with Eq. (10), $K = 0.2$. The correction

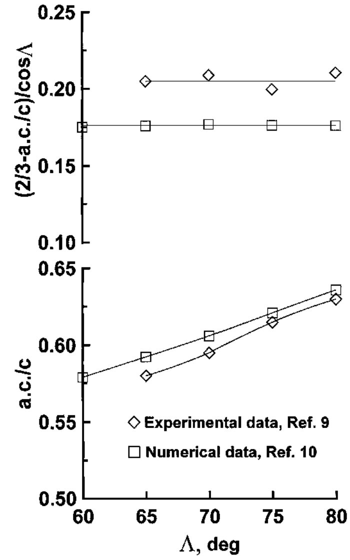

Fig. 1 Correlation of delta wing a.c. location as a function of Λ, α = 0 deg.

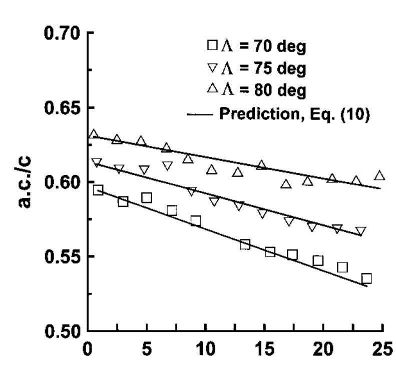

Fig. 2 Prediction of delta wing a.c. location as a function of α; experimental data from Ref. 9.

to Eq. (10), that is, $1 + 0.04\alpha$, does not embody any flow physics and is essentially a curve fit. However, Fig. 2 shows that Eq. (10) adequately predicts the behavior of the wing's a.c. with α, as well as its dependence on sweep. This expression should prove useful for conceptual design studies.

Substituting Eq. (9) into Eq. (5) gives

$$\frac{\Gamma(s)}{Uc(s)} = \frac{[c_s(s)/K]\left(\frac{2}{3} - \text{a.c.}/c\right)}{(1 - k_p k_i)\sin\alpha}$$

which on rearranging and solving for the sectional leading-edge suction (equal to the local vortex lift) becomes

$$c_s(s) = \frac{\Gamma(s)}{Uc(s)} \frac{K(1 - k_p k_i)\sin\alpha}{\left(\frac{2}{3} - \text{a.c.}/c\right)} \tag{11}$$

This expression suggests that the reduction in $\Gamma(s)$ with increasing sweep is counterbalanced by the rearward shift in the wing's a.c. location with increasing sweep. This implies that the relative invariance of vortex lift with increasing Λ is a result of reduced trailing-edge effects such that the net vortex lift coefficient remains relatively constant.

Conclusions

Polhamus's leading-edge suction analogy estimates that the vortex lift coefficient of delta wings is relatively insensitive to wing sweep. This is despite the reduction in vortex strength and increased vortex displacement from the wing surface that results from increasing sweep. An analytical investigation suggests that the invariance of the vortex lift coefficient is a result of increasing slenderness reducing trailing-edge effects. The analysis yields a simple explicit relationship between the a.c. and leading-edge sweep of a delta wing. This in turn allows the prediction of the a.c. and its variance with angle of attack for thin planar delta wings.

Acknowledgment

The author would like to thank Harry Hoeijmakers of the Department of Mechanical Engineering, Twente University, The Netherlands for his helpful comments.

References

[1]Polhamus, E. C., "A Concept of the Vortex Lift of Sharp-Edge Delta Wings Based on a Leading-Edge Suction Analogy," NASA TN D-3767, Oct. 1966.

[2]Traub, L. W., "Prediction of Delta Wing Leading-Edge Vortex Circulation and Lift-Curve Slope," *Journal of Aircraft*, Vol. 34, No. 3, 1997, pp. 450–452.

[3]Hemsch, M. J., and Luckring, J. M., "Connection Between Leading-Edge Sweep, Vortex Lift, and Vortex Strength for Delta Wings," *Journal of Aircraft*, Vol. 27, No. 5, 1990, pp. 473–475.

[4]Visser, K. D., and Nelson, R. C., "Measurements of Circulation and Vorticity in the Leading-Edge Vortex of a Delta Wing," *AIAA Journal*, Vol. 31, No. 1, 1993, pp. 104–111.

[5]Lowson. M. V., "Visualization Measurements of Vortex Flows," *Journal of Aircraft*, Vol. 28, No. 5, 1991, pp. 320–327.

[6]Purvis, J. W., "Analytical Prediction of Vortex Lift," *Journal of Aircraft*, Vol. 18, No. 4, 1981, pp. 225–230.

[7]Roos, F. W., and Kegelman, J. T., "An Experimental Investigation of Sweep-Angle Influence on Delta-Wing Flows," AIAA Paper 90-0383, Jan. 1990.

[8]Kirkpatrick, D. L. I., "Analysis of the Static Pressure Distribution on a Delta Wing in Subsonic Flow," Aeronautical Research Council, Research and Memoranda 3619, London, 1970.

[9]Traub, L. W., "Effects of Spanwise Camber on Delta Wing Aerodynamics: An Experimental and Theoretical Investigation," Ph.D. Dissertation, Aerospace Engineering Dept., Texas A&M Univ., College Station, TX, May 1999.

[10]Lamar, J. E., and Gloss, B. B., "Subsonic Aerodynamic Characteristics of Interacting Lifting Surfaces with Separated Flow Around Sharp Edges Predicted by a Vortex Lattice Method," NASA TN D-7921, Sept. 1975.

Control of a Three-Degree-of-Freedom Airfoil with Limit-Cycle Behavior

Robert L. Clark* and Earl H. Dowell†
Duke University, Durham, North Carolina 27708
and
Kenneth D. Frampton‡
Vanderbilt University, Nashville, Tennessee 37235

Introduction

LIMIT-CYCLE oscillations (LCOs) resulting from control surface freeplay are of concern in many aircraft because they typically occur at a dynamic pressure well below that of the linear flutter boundary. The stability and performance of the aeroservoelastic system is of particular importance in the presence of such nonlinearities that can develop during the life cycle of the aircraft. Results presented by Vipperman et al.[1] served to demonstrate that the control surface actuators can be used to provide successfully gust alleviation and extend the flutter boundary for a three-degree-of-freedom, linear, aeroelastic model. Additionally, Vipperman et al.,[2] as well as Frampton and Clark,[3] demonstrated that robust control strategies can be applied in the design of compensators for a family of dynamic pressures.

The purpose of this work is to investigate the effect of control surface freeplay nonlinearities on the closed-loop performance of a three-degree-of-freedom aeroelastic system. In particular, control systems designed for an open-loop linear three-degree-of-freedom system were applied to a nonlinear three-degree-of-freedom system and evaluated for their performance. It is vital that these linear compensators display stable, closed-loop response in the presence of freeplay nonlinearities that may evolve over the life cycle of the aircraft. Results from this study indicate that the limit-cycle amplitudes in both pitch and plunge can be attenuated significantly through the application of controllers designed for a linear three-degree-of-freedom aeroelastic system. The primary mechanism of control serves to convert high-amplitude, low-frequency LCOs to low-amplitude, high-frequency LCOs for the case considered.

In previous work the dynamic response of a three-degree-of-freedom aeroelastic typical section model with a single control surface extending over the span of the airfoil was investigated both analytically and experimentally.[4] In particular, control surface freeplay was investigated, and LCOs were observed. The freeplay nonlinearity was designed to produce a piecewise linear change in the structural stiffness of the control surface,[4] and the three-degree-of-freedom model was subjected to two-dimensional, incompressible flow. The development of the aeroelastic system follows that of Edwards et al.[5]

Three-Degree-of-Freedom System Model Description

As detailed by Conner et al.,[4] the three-degree-of-freedom model here is based upon the state-space model originally proposed by Edwards et al.[5] A schematic diagram of the model is depicted in Fig. 1. As illustrated, a flap control surface is attached to the wing, and a spring C_β provides a restoring force to the neutral position. The structural nonlinearity introduced for the purpose of this analysis is a spring with a symmetric freeplay region.

A block diagram of the dynamic system model is presented in Fig. 2. As illustrated, a structural nonlinearity is included in the

Received 20 May 1998; revision received 20 October 1999; accepted for publication 24 January 2000.

*Associate Professor of Mechanical Engineering and Materials Science, Box 90300.

†J.A. Jones Professor of Mechanical Engineering and Material Science, Box 90300.

‡Assistant Professor of Mechanical Engineering, Box 1592 Station B.